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Open String Moduli in KKLT Compactifications

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In the Kachru-Kallosh-Linde-Trivedi (KKLT) de-Sitter construction one introduces an anti-D3-brane that breaks the supersymmetry and leads to a positive cosmological constant. In this paper we investigate the open string moduli associated with this anti-D3-brane, corresponding to its position on the S^3 at the tip of the deformed conifold. We show that in the KKLT construction these moduli are very light, and we suggest a possible way to give these moduli a large mass by putting orientifold planes in the KKLT "throat".

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1. Introduction and Summary

Type II string compactifications to four spacetime dimensions with non trivial RR and NSNS background fluxes have been studied extensively in the literature in the past few years, as a way to stabilize moduli in string theory. Compactifications on generic Calabi-Yau three-folds without background fluxes lead to hundreds of massless scalar moduli fields, causing various phenomenological problems since no light scalar fields have been observed in nature. However, by turning on some background value for the fluxes on cycles of the Calabi-Yau, a potential develops that stabilizes those moduli at some fixed value and generates a mass for the scalar fields (see [1] and references therein).

Several examples of this mechanism, involving orientifolds, have been studied in detail. In type IIA string theory there are several known examples of toroidal orientifolds in which all moduli are stabilized. In type IIB string theory, the classical supergravity action generates a potential for the complex structure moduli of the Calabi-Yau manifold but not for its Kähler structure moduli. Since the total volume of the compact manifold is a Kähler modulus, it is not possible to fix all moduli by fluxes in the type IIB supergravity approximation. However, it has been argued [2] that non-perturbative effects in type IIB string theory, such as gauge theory instantons or gaugino condensation in the worldvolume of D7-branes or wrapped Euclidean D3-branes, generate a potential which depends also on the Kähler moduli. Including these nonperturbative effects leads to a potential with a minimum with a negative cosmological constant, describing a supersymmetric Anti de-Sitter (AdS) background.

The authors of [2] suggested that a slight modification of such a background could lead to a meta-stable de Sitter (dS) background, in agreement with recent observations suggesting a positive cosmological constant. The modification involves introducing a space-filling anti-D3-brane (which we will denote as a $\overline{\mathrm{D3}}$-brane) which raises the potential energy. This breaks all the supersymmetry, and using some fine tuning it was argued that it is possible to obtain a positive yet small cosmological constant. Following the work of [2], various other suggestions for constructing meta-stable dS vacua have also appeared.

In addition to changing the potential, the addition of the $\overline{\mathrm{D3}}$ brane has implications regarding the moduli in the theory. In the presence of the $\overline{\mathrm{D3}}$ brane there is also an open string sector, which includes some light scalar fields (moduli) that can be interpreted as the location of the $\overline{\mathrm{D3}}$ brane in the compact space. In this paper we study these moduli.

We begin in section 2 by reviewing the KKLT construction, in which the moduli are stabilized near a conifold singularity such that the compactification includes a Klebanov-Strassler (KS) [3] type "throat", generating a hierarchy by a factor of the small warp factor a_0 at the tip of the "throat" [4], and a $\overline{\mathrm{D3}}$ brane is then added at the tip of the "throat". In section 3 we discuss the mass of the open string moduli corresponding to the position of the $\overline{\mathrm{D3}}$ brane. We argue that in the limit of an infinite "throat" these moduli are massless since they are Goldstone bosons, but when the "throat" is finite the background is changed and the moduli obtain a mass. We discuss in detail the deviation of the finite "throat" theory from the infinite "throat" theory of [3], and we identify the leading deviation which contributes to the mass of the open string moduli. We use the approximate conformal symmetry of the "throat" theory to classify the deviations, and we find that the leading deviation corresponds to an operator of dimension $\Delta = \sqrt{28} \simeq 5.29$, and that it leads to a mass squared for the open string moduli scaling as $a_0^{\Delta-2} \simeq a_0^{3.29}$. In the interesting limit of large warping, $a_0 \ll 1$, this mass is exponentially lighter than the other mass scales appearing in the warped compactification, implying that the KKLT scenario generally leads to light scalars which could cause phenomenological problems.

In section 4 we suggest a possible way to resolve this problem and increase the mass of the moduli, by positioning two of the orientifold 3-planes (which must be present anyway in KKLT-type compactifications) at the tip of the "throat", and adding to them half-D3-branes so that they become $\mathrm{O3}^+$-planes rather than $\mathrm{O3}^-$ planes. The $\overline{\mathrm{D3}}$ brane is then attracted to these $\mathrm{O3}^+$ planes, increasing the mass of the open string moduli. The mass squared is still smaller than the typical mass scales, but only by a factor of the string coupling g_s which does not have to be very small, so this may not lead to phenomenological problems (especially if the standard model fields live in a different position in the Calabi-Yau and couple very weakly to the $\overline{\mathrm{D3}}$ brane fields). Our scenario has the added advantage that by adding two half D3-branes in addition to the $\overline{\mathrm{D3}}$ brane we do not generate a tadpole for the D3-brane charge, unlike the original KKLT scenario where such a tadpole exists and leads to subtleties in using the probe approximation for describing the $\overline{\mathrm{D3}}$ brane (due to the necessity to change the background elsewhere to compensate for the $\overline{\mathrm{D3}}$ brane charge).

Finally, in two appendices we derive some results used in the text. In appendix A we list the possible deformations of the $AdS_5 \times T^{1,1}$ background (which is a good approximation to the "throat") which can appear as deformations of the "throat" in our background. In appendix B we discuss the moduli space of the gauge theory dual to the "throat" region after deformations by superpotential operators, and we argue that any such deformations reduce the dimension of the moduli space.

2. A review of dS flux compactifications with $\overline{\mathrm{D3}}$-branes

The setting for our analysis in the following sections is the dS background of KKLT [2]. We start with a brief overview of a general flux compactification and then proceed to describe the construction of the dS background. More details can be found in [2,5,4].

2.1. Warped flux compactifications

We consider type IIB string theory in the supergravity approximation, described in the Einstein frame by the action

$$
\begin{aligned}
S_{IIB} =& \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left\{\mathcal{R} - \frac{\partial_M\tau\partial^M\bar{\tau}}{2(\mathrm{Im}\tau)^2} - \frac{G_3\cdot\bar{G}_3}{12\mathrm{Im}\tau} - \frac{\tilde{F}_5^2}{4\cdot 5!}\right\} \\
&+ \frac{1}{8i\kappa_{10}^2}\int\frac{C_4\wedge G_3\wedge G_3}{\mathrm{Im}\tau} + S_{local},
\end{aligned}
\tag{2.1}
$$

where $\tau = C_0 + ie^{-\phi}$ is the axio-dilaton field and we combine the RR and NS-NS three-form fields into the generalized complex three-form field $G_3 = F_3 - \tau H_3$. In addition one must impose a self duality condition on the five form $\tilde{F}_5 \equiv F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3$,

$$
\tilde{F}_5 = *\tilde{F}_5. \tag{2.2}
$$

The local action S_{local} includes the contributions from additional local objects such as D-branes or orientifold planes.

We begin by considering warped backgrounds, with a metric of the form

$$
ds_{10}^2 = \mathrm{e}^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + \mathrm{e}^{-2A(y)}\tilde{g}_{mn}(y)dy^m dy^n, \tag{2.3}
$$

where $\mu,\nu = 0,1,2,3; m,n = 4,\cdots,9$, and the unwarped metric $\tilde{g}_{mn}$ scales as $\sigma^{1/2}$, where σ is the imaginary component of the complex Kähler modulus related to the overall scale of the compact Calabi-Yau. In addition, both the five form and three form fields are turned on. Due to 4-dimensional Poincaré invariance only compact components of G_3 may be turned on, while for the five form, the Bianchi identity determines it to be of the form

$$
\tilde{F}_5 = (1+*)d\alpha(y)\wedge dx^0\wedge dx^1\wedge dx^2\wedge dx^3. \tag{2.4}
$$

Finally, local objects extended in the four non-compact dimensions can be added wrapping cycles of the compact space. These must satisfy the tadpole cancellation condition

$$
\frac{1}{2\kappa_{10}^2 T_3}\int_{\mathcal{M}_6} H_3\wedge F_3 + Q_3^{local} = 0, \tag{2.5}
$$

where Q_3^{local} is the D3-brane charge of the local objects.

The supergravity equations of motion for such a configuration of fields can be conveniently written in terms of the following combinations of the five form and warp factor

$$\Phi_{\pm} \equiv e^{4A} \pm \alpha. \tag{2.6}$$

The Einstein equation and the Bianchi identity for the 5-form field can be combined to give

$$\tilde{\nabla}^2 \Phi_{\pm} = \frac{e^{2A}}{6\mathrm{Im}\tau} |G_{\pm}|^2 + e^{-6A} |\nabla \Phi_{\pm}|^2 + local, \tag{2.7}$$

where we defined the imaginary self dual (ISD) and imaginary anti self dual (IASD) components of the generalized three form flux,

$$G_{\pm} = iG \pm *_6 G \qquad \Rightarrow \qquad *_6 G_{\pm} = \pm i G_{\pm}. \tag{2.8}$$

The local objects act as sources for the fields $\Phi_{\pm}$. D3-branes and O-planes appear as sources only in the equation for Φ_+, while $\overline{\text{D3}}$-branes appear only in the equation for Φ_-. For a background with no $\overline{\text{D3}}$-branes there are no sources for Φ_-, so we get using (2.7) and the compactness of the Calabi-Yau

$$\Phi_- = 0 \Rightarrow \alpha = e^{4A}. \tag{2.9}$$

Since $|G_-|^2$ is positive definite it must vanish everywhere and so G_3 is ISD.

The equations of motion can be compactly summarized by a 4-dimensional superpotential [5]

$$W = \int \Omega \wedge G_3, \tag{2.10}$$

where Ω is the holomorphic $(3,0)$ form, together with the standard supergravity Kähler potential. This notation makes explicit the fact that the equations give non-trivial restrictions on some of the moduli. The superpotential depends both on the axio-dilaton (through its appearance in G_3) and on the geometrical complex structure moduli that appear in Ω. However, the resulting four dimensional supergravity theory is of the no scale class. The Kähler moduli, including the global volume of the compact manifold, have no potential (in the supergravity approximation) and remain unfixed.

Consider probing this space with D3-branes. The D3-brane action in the Einstein frame without turning on any open string fields, including both the DBI and the Wess-Zumino term, is given by

$$S_{\mathrm{D3}} = -T_3 \int \sqrt{g_4} d^4x \Phi_-. \tag{2.11}$$

For the type of solutions discussed above, obeying equation (2.9), we obtain that these probes feel no force, and their moduli space is the full compact manifold. For $\overline{\mathrm{D3}}$-brane probes, due to the opposite sign in the Wess-Zumino term, we find that the action is

$$S_{\overline{\mathrm{D3}}} = -T_3 \int \sqrt{g_4} d^4x \Phi_+. \tag{2.12}$$

In our background where $\Phi_+ = 2e^{4A}$ there is thus a force on the $\overline{\mathrm{D3}}$-brane driving it towards smaller values of the warp factor.

2.2. Getting a hierarchy from the conifold

It is phenomenologically interesting to find a background in which, in addition to fixing the moduli, there is a large warped throat. This can be used to realize the construction of Randall and Sundrum [6,7,8], giving a solution to the hierarchy problem. Such a background was found in [4] by considering a generic Calabi-Yau near a special point in its moduli space where it develops a singularity. Generically such a singularity looks locally like the conifold singularity [9] which can be described by the sub-manifold of $\mathbb{C}^4$ defined by:

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0. \tag{2.13}$$

The conifold is a cone whose base is $T^{1,1} = (SU(2) \times SU(2))/U(1)$, a fibration of S^3 over S^2. The cone is singular at $(z_1, z_2, z_3, z_4) = (0, 0, 0, 0)$ where the spheres shrink to zero size. The isometry group of the base geometry is easily seen to be $SU(2) \times SU(2) \times U(1)$, where the $SU(2) \times SU(2) \simeq SO(4)$ rotates the z_i's and the $U(1)$ adds a constant phase $z_i \to e^{i\alpha} z_i$.

The singularity of the conifold can be smoothed in two ways, by blowing up either of the spheres to a finite size. We will be interested in the deformation of the conifold, which is the sub-manifold given by

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = \mu, \tag{2.14}$$

where μ becomes a complex structure modulus for this manifold. Geometrically, in (2.14) the S^2 shrinks to zero size at the tip while the S^3 remains at some finite size. The minimal size S^3 at the "tip" is given by

$$|z_1|^2 + |z_2|^2 + |z_3|^2 + |z_4|^2 = |\mu|. \tag{2.15}$$

This deformation breaks the symmetry group to $SU(2) \times SU(2) \times \mathbb{Z}_2$, where the $SO(4)$ can be understood geometrically as rotations of the S^3.

Placing M fractional D3-branes at a conifold singularity, the background near the singularity is given, for large g_sM and for some range of radial distances from the singularity, by the KS solution [3], where in the near horizon geometry one replaces the branes by fluxes. Such a configuration involves turning on M units of F_3 flux on the S^3 at the tip of the conifold, and also $(-K)$ units of H_3 flux on the dual cycle (which is non-compact in [3] but is compact when we embed this into a compact Calabi-Yau). It is customary to define $N = MK$. In [4] it was found that such fluxes generate a warped throat similar to [3] near the singularity. The superpotential stabilizes the complex structure modulus μ at a value for which the warp factor at the tip of the throat is given by

$$a_0 \equiv e^{A_0} = e^{-2\pi K/3Mg_s}, \tag{2.16}$$

which is exponentially small when $K \gg g_sM$ (the validity of the supergravity approximation in the "throat" requires also $g_sM \gg 1$).

In the throat region of the Calabi-Yau the warp factor is given by

$$e^{-4A} = \frac{27\pi}{4u^4}\alpha'^2 g_s N \left(1 + \frac{g_sM}{K}\left(\frac{3}{8\pi} + \frac{3}{2\pi}\ln(\frac{u}{u_0})\right)\right) \tag{2.17}$$

where u is the radial coordinate along the throat. At the tip of the throat the redshift is minimal and given by (2.16). There we get $u \sim Ra_0$, where we defined $R^4 = \frac{27}{4}\pi\alpha'^2 g_s N$. The bulk of the Calabi-Yau, where the warp factor is of order unity (and deviations from (2.17) are large) is at $u \sim R$.

2.3. Lifting to a dS background

Although phenomenologically interesting, backgrounds of this type classically have at least one scalar modulus. The low-energy theories we arrive at are no-scale models, and the potential generated by the fluxes does not give any mass term for the Kähler modulus related to the volume of the compact space. This was mended in [2] by considering non-perturbative effects. Terms in the potential coming either from instantons in non-Abelian gauge groups on a stack of D7-branes or from Euclidean D3-branes wrapped on 4-cycles depend on the volume of the space, and stabilize it at some finite value.

The stabilization of the Kähler modulus leads to a vacuum with a negative cosmological constant, an AdS space. It was then argued that adding an $\overline{\text{D3}}$-brane (which, as discussed above, should sit at the "tip" of the throat to be stable) results in a positive contribution to the scalar potential from (2.12), and with some tuning of the parameters it can lift the minimum of the potential to a small positive value. Thus it is possible to get a de-Sitter space with a small cosmological constant.

3. The $\overline{\text{D3}}$-brane moduli

A consequence of the introduction of an $\overline{\text{D3}}$-brane to the warped background is the addition of new light scalar fields from the open strings ending on the $\overline{\text{D3}}$-brane, corresponding to the position of the $\overline{\text{D3}}$-brane on the compact space. In this section we analyze the potential for these moduli in the KKLT background. We first consider the background without the additional $\overline{\text{D3}}$-brane, and estimate the deviation of the warped background with a compact Calabi-Yau from the non-compact background of [3]. We then use this to estimate the masses for the position of the $\overline{\text{D3}}$-brane, using the action (2.12) and considering the $\overline{\text{D3}}$-brane as a probe (as in [2]). This approximation is valid when $g_s \ll 1 \ll g_s M$.

From (2.12) we see that the $\overline{\text{D3}}$-branes are not free to move on the compact space since they have a non trivial potential proportional to the warp factor. This potential drives them to the tip of the throat where the warp factor is minimal, giving a mass to the scalar field corresponding to the radial position of the $\overline{\text{D3}}$-brane.

At the tip of the throat, the $\overline{\text{D3}}$-brane can still move on the S^3. In the full infinite KS solution there is an exact $SO(4)$ symmetry corresponding to rotations in this 3-sphere, and placing the $\overline{\text{D3}}$-brane breaks this symmetry as $SO(4) \to SO(3)$. This gives rise to three massless moduli, the three Goldstone bosons, which can also be interpreted as the three coordinates of the position of the $\overline{\text{D3}}$-brane in the S^3.

In our background there are, however, corrections coming from the compactness of the Calabi-Yau, as the background deviates from the KS solution away from the tip. From the point of view of the field theory dual of the KS background, these corrections are related to UV perturbations (changes in coupling constants). Some of these corrections explicitly break the $SO(4)$ symmetry, and thus generate a mass for the Goldstone bosons. We will first classify the possible perturbations that can be turned on in this class of backgrounds, and then go on to consider their effect on the mass for the three moduli of the $\overline{\text{D3}}$-brane.

3.1. UV corrections of the background

The deformation of our background away from the KS geometry, at large radial position away from the conifold, is easily described in the language of the dual field theory. The dual theory (at some cutoff scale) has an $SU(N) \times SU(N+M)$ gauge group, with gauge superfields W_1 and W_2 corresponding to the two gauge groups, and two doublets of chiral superfields A_i, B_i $(i = 1,2)$ in the $(N, \overline{N+M})$ and $(\overline{N}, N+M)$ representations, respectively, of the $SU(N)\times SU(N+M)$ group, and in the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations of the global $SU(2) \times SU(2)$ symmetry.

In the dual description the region near the singularity describes the low-energy physics of the field theory while the Calabi-Yau end of the throat serves as a UV cutoff of the field theory. Deforming the solutions at large radial position is described by changing the theory at some large UV scale where the effective theory is some deformation of the KS theory,

$$\mathcal{L} = \mathcal{L}_{KS} + c_i \int \mathcal{O}_i. \tag{3.1}$$

Generally all possible operators might be turned on at this scale, and they could influence the $\overline{\mathrm{D3}}$-brane at the tip (the IR limit) and give a mass to the moduli. Due to the renormalization group flow the contribution to the mass of the $\overline{\mathrm{D3}}$-brane at the tip will be dominated by the most relevant operators at the IR, namely the lowest dimension operators. Relevant and marginal operators will have a large effect, while that of the irrelevant operators will be suppressed.

It is sufficient to analyze the operators and their dimensions in the conformal case [10] where the gauge group is $SU(N) \times SU(N)$, since the cascading case is expected to behave similarly up to log corrections and operator mixings which should not change our conclusions. For this case the classification of all supergravity KK-modes on $T^{1,1}$ and the corresponding operators in the field theory was given in [11,12]. Since we are only interested in turning on operators that break neither 4-dimensional Lorentz invariance nor supersymmetry, we can restrict our attention to the highest components of the different superconformal multiplets and consider only those that are Lorentz scalars. The only possible operators come from vector multiplets of the five dimensional gauged supergravity which arises by KK reduction on $T^{1,1}$, either long multiplets or chiral multiplets.

The analysis of supergravity modes is carried out in appendix A, where we find only one possible relevant operator

$$S_1 = \int d^2\theta \ \mathrm{Tr}(A^i B^j), \qquad i,j = 1,2, \qquad \Delta_{S_1} = 2.5, \tag{3.2}$$

and three possible marginal operators

$$\begin{aligned} S_2 &= \int d^2\theta \ \mathrm{Tr}(A^i B^j A^k B^l), \qquad \Delta_{S_2} = 4, \\ \Phi_0 &= \int d^2\theta \ \mathrm{Tr}(W_1^2 + W_2^2), \qquad \Delta_{\Phi_0} = 4, \\ \Psi_0 &= \int d^2\theta \ \mathrm{Tr}(W_1^2 - W_2^2), \qquad \Delta_{\Psi_0} = 4. \end{aligned} \tag{3.3}$$

The operator S_2 is symmetric in (i,k) and (j,l); the anti-symmetric combination mixes with Φ_0. There is also an infinite number of irrelevant operators, all of them with dimensions $\Delta \geq 5.29$.

In fact not all possible operators are turned on in the compact Calabi-Yau background. As discussed above, a probe D-brane in this background must feel no force and its moduli space should describe the full 6 dimensional compact geometry. In appendix B it is found that the addition of the operators S_1, S_2 changes the moduli space drastically and necessarily results in a force on the D3-brane. Thus, these operators are not turned on in the warped flux compactifications.

The two marginal operators, Φ_0 and Ψ_0, can be turned on, but they are symmetric under the $SU(2)\times SU(2)$ and do not lead to symmetry breaking and to a mass for the $\overline{\text{D3}}$-brane moduli. From the field theory perspective they correspond to changing the coupling constants that are already present in the non-deformed theory and do not generate new terms in the action.

3.2. Masses from UV corrections

In the previous subsection we have seen that relevant operators are not turned on in the warped background, while the possible marginal operators do not break the symmetry and leave the moduli massless. Irrelevant operators, however, can be turned on, and we next discuss the masses generated by them. As discussed in appendix A, the various operators which preserve SUSY and Lorentz invariance are related to Kaluza-Klein modes of the warped metric on the $T^{1,1}$ $\hat{g}_{ij}$, the field Φ_+ defined in (2.6), the three form field G_3 and the axio-dilaton τ.

The operators are turned on at the UV cutoff, and in order to consider their effect on the IR physics we need to discuss their flow, or in the supergravity language their profile along the radial coordinate. We start by considering the profiles of the fields corresponding to these operators on $AdS_5 \times T^{1,1}$, using the metric $ds^2_{AdS} = u^2 dx^\mu dx_\mu + du^2/u^2$. There are two independent solutions for the field ϕ corresponding to an operator of dimension $\Delta > 2$, with the following u-dependence :

$$\phi(u) = a u^{-\Delta} + b u^{\Delta - 4}. \qquad (3.4)$$

In the KS background there are small logarithmic corrections to this, and in addition the behavior near the tip of the "throat" gives some IR boundary condition for the field

equations. Generically this implies that at $u = Ra_0$ the two terms are of the same order. Then, at the UV cutoff $u \sim R$ (the Calabi-Yau), the second term will dominate so

$$\phi(u \sim R) \simeq bR^{\Delta-4}. \tag{3.5}$$

Deforming the theory at the UV by some $\delta\phi(R) \sim \phi_0$ will then correspond in the IR to

$$\delta\phi(Ra_0) \sim \phi_0 \frac{(Ra_0)^{\Delta-4}}{R^{\Delta-4}} = \phi_0 a_0^{\Delta-4}. \tag{3.6}$$

The deformation in the IR is suppressed for operators with higher dimension, as expected.

The largest contribution to a mass of an object localized near the tip will be from the operator with lowest dimension that breaks the $SU(2) \times SU(2)$ global symmetry. The analysis of the previous subsection and Appendix A implies that this is the lowest component of the vector multiplet I, with $j = l = 1$ and $r = 0$, whose dimension is $\Delta = \sqrt{28} \simeq 5.29$. This operator corresponds in the supergravity to a KK mode of the warped metric $\hat{g}_{ij}$. We do not see any reason why this operator should not appear in the CY compactification so we assume that it does[4]. At the UV we have $\hat{g}_{ij} \sim \sigma^{1/2}$, and we expect the deformation of the metric to be of the same order as the metric so we can approximate $\delta\hat{g}_{ij}|_{\mathrm{UV}} \sim \sigma^{1/2}$ and

$$\delta\hat{g}_{ij}|_{\mathrm{IR}} \sim \sigma^{1/2} a_0^{\Delta-4} = \sigma^{1/2} a_0^{1.29}. \tag{3.7}$$

In order to evaluate the corresponding mass we need to write in more detail the action on a probe $\overline{\mathrm{D3}}$-brane. We consider a $\overline{\mathrm{D3}}$-brane filling the non compact space-time and positioned at the point X^m in the compact Calabi-Yau, which is near the tip of the throat. Expanding around this position, we get the action (2.12) with an additional kinetic term. Using the solution of the supergravity equations for the five form (2.9) we get

$$S_{\overline{\mathrm{D3}}} = -2T_3 \int \sqrt{g_4} d^4x e^{4A(X^m)} - T_3\alpha' \int \sqrt{g_4} d^4x \frac{1}{2} g_4^{\mu\nu} \partial_\mu X^m \partial_\nu X^n \tilde{g}_{mn}, \tag{3.8}$$

where $\tilde{g}_{mn}$ is the unwarped metric, $\tilde{g}_{mn} \simeq a_0^2 \hat{g}_{mn}$. Mass terms appear in this action only through the dependence of the warp factor, e^{4A}, on the position X^m. Since in the full non-compact case the warp factor has only radial dependence, for a mass to be generated

[4] Note that this operator deforming the throat seems to be different from the one analyzed in the appendix of [13].

in the S^3 directions we need to consider the change in the warp factor due to the deformed supergravity fields $\hat{g}_{ij}$.

Tracing the Einstein equation and using (2.9) we can write the equation of motion for the warp factor as

$$\hat{\nabla}^2 A = \frac{g_s}{48}|G|^2, \tag{3.9}$$

where $\hat{\nabla}^2$ is the Laplacian on the warped compact space and we use the warped metric to raise and lower indices. The change in A due to the deformation in $\hat{g}_{ij}$ will thus satisfy

$$\hat{\nabla}^2 \delta A = \frac{g_s}{48} G_{m_1 n_1 i} G^*_{m_2 n_2 j} \hat{g}^{m_1 m_2} \hat{g}^{n_1 n_2} \delta \hat{g}^{ij} \sim a_0^{\Delta - 4}. \tag{3.10}$$

In this equation we dropped a term $(\delta \hat{\nabla}^2) A$ since the change in the Laplacian due to the deformation in the compact metric $\hat{g}_{ij}$ will be proportional to derivatives in those directions, while the original A has no such dependence and so this term vanishes.

The masses arise due to the variation of A in the S^3 at the tip, where the $\overline{\mathrm{D3}}$-brane position is parameterized by X^i. Since in the warped metric we are using, this 3-sphere has constant size (with a radius $\sim \sqrt{g_s M}$), we can estimate

$$A \sim A_0 + (g_s M)^{-1} a_0^{\Delta - 4} \hat{g}_{ij} X^i X^j. \tag{3.11}$$

Plugging into (3.8) we find

$$S_{\overline{\mathrm{D3}}} \sim -T_3 \int \sqrt{g_4} d^4x \left[2a_0^4 + 2(g_s M)^{-1} a_0^{\Delta - 2} \tilde{g}_{ij} X^i X^j + \frac{\alpha'}{2} g_4^{\mu\nu} \partial_\mu X^m \partial_\nu X^n \tilde{g}_{mn} \right] \tag{3.12}$$

where we changed the metric to the unwarped metric in both kinetic and mass terms[5]. We see that a mass term was generated with a mass of the order of $m^2 \sim (g_s M \alpha')^{-1} a_0^{\Delta-2} = (g_s M \alpha')^{-1} a_0^{3.29}$.

We see that the deformation of the theory at the UV does indeed generate mass terms for the open string moduli. However, the highest contribution is of order $a_0^{3.29}$. In the warped background the typical IR mass scale is of order a_0^2, so the mass generated here is exponentially smaller (given (2.16)). In the Klebanov-Strassler background there are presumably subleading logarithmic corrections to this result, however it is still highly suppressed. Such light moduli would lead to phenomenological problems if we try to use such a scenario to describe the real world.

[5] Note that the first term, even though naively it is independent of the volume σ of the compact space, actually does give a potential for the volume factor [2] when we rescale the four dimensional metric canonically [14].

4. A large open string moduli mass from O-planes

It is possible to obtain a higher mass for the open string moduli by using O-planes. In this section we calculate this mass. Recall that integrating the supergravity equation of motion (2.7) on the compact space we get that the left hand side vanishes since there are no boundaries. The right hand side is positive definite, except for possible negative contributions in the local terms corresponding to orientifold planes. Hence in general we must have orientifold 3-planes in order to be able to solve the equations of motion. It is then natural to try and use these orientifolds for the purpose of stabilizing the moduli for the $\overline{\mathrm{D3}}$-brane, by choosing the position of these orientifolds to be at the tip of the throat.

For simplicity we consider an orientifold of the non-compact Klebanov-Strassler solution. Since the analysis is local, embedding this into the full background will not change the conclusions. The action of the orientifold is defined as in [15] by

$$(z_1, z_2, z_3, z_4) \to (z_1, -z_2, -z_3, -z_4). \tag{4.1}$$

This orientifold has two fixed points, both on the tip of the deformed conifold (2.15) at the poles of the S^3, $(z_1, z_2, z_3, z_4) = (\pm\sqrt{\mu}, 0, 0, 0)$. Physically there are two O3-planes at these points, which will interact with the $\overline{\mathrm{D3}}$-brane and generate a potential for its position on the 3-sphere. Note that the addition of the O3-planes has no effect on the supersymmetry of the model, since the O3-planes break the same supercharges as the fluxes.

The D3 charge of an orientifold plane as well as its tension is negative (equal to $-1/4$ that of a D3-brane), while for $\overline{\mathrm{D3}}$-branes the charge is negative and the tension is positive. We see that both effects result in a repulsive force, so that the $\overline{\mathrm{D3}}$-brane does not get stabilized but rather it would want to sit on the equator of the S^3. However one can use half-D3-branes to fix the situation. Putting a half-D3-brane on the orientifold singularity, we get an $\mathrm{O3}^+$-plane with the opposite charge and tension. Since we have two singular points we can add two such half-D3-branes to make both O-planes positively charged. Note that the insertion of one additional unit of D3-brane charge is actually a necessity once we introduce the $\overline{\mathrm{D3}}$-brane, due to the tadpole cancellation condition. Assuming that without $\overline{\mathrm{D3}}$-branes the background with $\mathrm{O3}^-$ planes is a solution of the supergravity, inserting the $\overline{\mathrm{D3}}$-brane will cause a deficiency in D3 charge, which can be resolved by the extra two half D3-branes. Note that an $\overline{\mathrm{D3}}$-brane cannot annihilate with a half-D3 so the solution should still be (meta)-stable.

The potential between the $\overline{\text{D3}}$-brane and the O3$^+$-plane can be calculated by world-sheet methods [16]. The first contribution which depends on the distance between the $\overline{\text{D3}}$-brane and the orientifold comes from the Möbius strip, and is equal to

$$\mathcal{M} = 2T_3 g_s \sum_{n=0}^{\infty} c_n r^{2n}, \tag{4.2}$$

where r is the distance (in string units) between the $\overline{\text{D3}}$-brane and the O3$^+$-plane, namely $r^2 = \hat{g}_{ij} X^i X^j$, and the coefficients are

$$c_n = (-1)^{n+1} k_{n-3} \left(\frac{2^{n-4} \pi^{(2-n)/2}}{n!} \right), \tag{4.3}$$

where k_{n-3} is a positive number for $n = 0, 1$ [16]. This computation was done in flat space, but for large $g_s M$ the curvature is small and it is a good approximation.

The first term in the expansion is a correction to the energy which is independent of the distance. The second term is a quadratic potential for the position of the $\overline{\text{D3}}$-brane which describes attraction between the $\overline{\text{D3}}$-brane and the O3$^+$-plane. The contribution to the action is

$$-2T_3 \int \sqrt{g_4} d^4x a_0^4 g_s c_1 \hat{g}_{ij} X^i X^j = -2T_3 \int \sqrt{g_4} d^4x a_0^2 g_s c_1 \tilde{g}_{ij} X^i X^j, \tag{4.4}$$

so that the mass of the X^i fields is

$$m^2 \sim \frac{2c_1}{\alpha'} g_s a_0^2. \tag{4.5}$$

We see that the orientifold gives these fields a mass of order $m^2 \sim g_s a_0^2$, which is the same scale as generic low mass scales in this background. Since this mass comes from the Möbius strip, it is suppressed by a factor of g_s compared to other masses so these open string moduli are still light, but not exponentially as before, so hopefully they should not cause phenomenological problems.

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Appendix A. Low dimensional operators from supergravity analysis

The Kaluza-Klein spectroscopy for the supergravity fields on $AdS_5 \times T^{1,1}$ was carried out in [11,12]. In this section we will review their results for the dimensions of the corresponding operators as a function of their $SU(2) \times SU(2) \times U(1)_r$ quantum numbers j, l, r. By considering the group theoretic restrictions on these quantum numbers for each multiplet, we will be able to find the operators with lowest dimension.

In general we expand the fields in spherical harmonics on the 5 dimensional compact space $T^{1,1} = \frac{SU(2)\times SU(2)}{U(1)}$, with fields of different spins on the compact space expanded using the corresponding $SO(5)$ harmonics. These harmonics furnish representations of the isometry (global symmetry) group, $SU(2) \times SU(2) \times U(1)_r$ in our case, but not all representations appear in the expansion. The specific participating representations depend on the Lorentz properties of the fields, but it turns out that all representations satisfy that either both $SU(2)$ spins j and l are integers or both are half integers.

All resulting modes can be arranged into multiplets of the $\mathcal{N} = 1$ $d = 4$ superconformal algebra. There are nine types of multiplets – one graviton multiplet, four gravitini and four vector multiplets [11,12]. For specific values of the quantum numbers, some multiplets obey a shortening condition and become semi-long, massless or chiral multiplets.

For the current analysis we are interested in operators that can be turned on at some UV cut-off without breaking four dimensional Lorentz invariance or supersymmetry. Hence the relevant multiplets are only those with scalars as the highest component. These are only the vector multiplets, either with generic values of the quantum numbers or when they obey the condition for shortening to chiral multiplets. The top components of these multiplets are related to Kaluza-Klein modes of the warped 5D metric on the $T^{1,1}$ $\hat{g}_{ij}$, the field Φ_+ defined in (2.6), the three form field G_3 and the axio-dilaton τ.

The first vector multiplet (vector multiplet I in the notations of [11,12]) has a top component related to the Kaluza-Klein modes of the warped 5D metric on the $T^{1,1}$ $\hat{g}_{ij}$, both when it is long and when it obeys the chiral shortening condition. The dimension of this multiplet, defined as the dimension of the lowest component, is given by

$$\Delta = \sqrt{H(j,l,r)+4} - 2, \tag{A.1}$$

where

$$H(j,l,r) \equiv 6(j(j+1) + l(l+1) - \frac{r^2}{8}), \tag{A.2}$$

with (j, l, r) the quantum numbers for the representation of the $SU(2) \times SU(2) \times U(1)_r$ symmetry group. The lowest component b, coming from a linear combination of the 5-form and the warp factor $\Phi_- = e^{4A} - \alpha$, is expanded in scalar harmonics that satisfy that r is even (odd) for j, l integers (half integers) and $|r| \leq 2\min(j, l)$.

Small dimensions arise when H is small. Due to the 1/8 factor in the third term, large values of j and l cannot be compensated by large values of r and will give higher values. It is then enough to look at small values for j and l. The lowest values and corresponding quantum numbers are written in table 1. In addition, for each multiplet one can check whether it obeys some shortening condition and what is the dimension of the operator corresponding to the top component. The $j = l = r = 0$ chiral operator can in fact be gauged away, so among the physical scalar operators we are left with one relevant operator and one marginal operator, and all others are irrelevant.

j	l	$\|r\|$	H	Δ	Type	Δ_{top}
0	0	0	0	0	chiral	1
1/2	1/2	1	8.25	1.5	chiral	2.5
0	1	0	12	2	semilong	–
1	0	0	12	2	semilong	–
1	1	2	21	3	chiral	4
1	1	0	24	3.29	none	5.29
1/2	3/2	1	26.25	3.5	semilong	–
3/2	1/2	1	26.25	3.5	semilong	–

Table 1: *Lowest dimensional operators from vector multiplet I.*

A similar analysis can be done for vector multiplet II for which the top component is related to Φ_+. This multiplet does not satisfy any shortening condition. The dimension of the multiplet is given by a similar expression

$$\Delta = \sqrt{H(j, l, r) + 4} + 4. \tag{A.3}$$

In this case the top component is itself a mode of a ten dimensional scalar field so the quantum numbers satisfy the same inequality as in the previous case. The lowest dimensional operator has $H = 0 \to \Delta = 6 \to \Delta_{top} = 8$ which is already irrelevant. Some of the low dimensional operators are described in table 2.

j	l	$\|r\|$	H	Δ	Type	Δ_{top}
0	0	0	0	6	none	8
1/2	1/2	1	8.25	7.5	none	9.5
0	1	0	12	8	none	10
1	0	0	12	8	none	10
1	1	2	21	9	none	11
1	1	0	24	9.29	none	11.29
1/2	3/2	1	26.25	9.5	none	11.5
3/2	1/2	1	26.25	9.5	none	11.5

Table 2: *Lowest dimensional operators from vector multiplet II.*

For the vector multiplet III, the top component (whether or not the multiplet obeys a shortening condition) is related to the three form field G_3, and the dimension of the multiplet is

$$\Delta = \sqrt{H(j, l, r + 2) + 4} + 1. \tag{A.4}$$

For this multiplet none of the fields are expanded in scalar harmonics. Instead, the top component a (which is the same for both the long and chiral multiplets) originating from the ten dimensional two-form potential is expanded using the two-form harmonics. For these harmonics we again have that r is even (odd) for j, l integers (half integers), but now the restriction on the quantum numbers is $|r| \leq 2\min(j, l) + 2$.

In this case we also have non trivial restrictions from the unitarity bounds

$$2 - \Delta \leq \frac{3}{2}r \leq \Delta - 2. \tag{A.5}$$

The possible quantum numbers are given in table 3.

j	l	r	H	Δ	Type	Δ_{top}
0	0	0	-3	2	chiral	4
1/2	1/2	1	2.25	3.5	chiral	5.5
1/2	1/2	-1	8.25	4.5	none	6.5
0	1	0	9	4.61	none	6.61
1	0	0	9	4.61	none	6.61

Table 3: *Lowest dimensional operators from vector multiplet III.*

Finally, vector multiplet IV has dimension given by

$$\Delta = \sqrt{H(j,l,r-2)+4}+1. \tag{A.6}$$

For long multiplets the top component is related to the three form field G_3 while for multiplets satisfying a chiral shortening condition the top component is related to the axio-dilaton τ. This last mode appear in all vector multiplets of this type (though it is not always the top component) and it is expanded in scalar harmonics. Its r-charge equal to $r-2$ so the quantum numbers obey $|r-2| \leq 2\min(j,l)$. Since the dimension depends only on $r-2$ we get a similar table to that of vector multiplet I but with the dimensions shifted:

j	l	r	H	E_0	Type	Δ_{top}
0	0	0	0	3	chiral	4
1/2	1/2	1	8.25	4.5	chiral	5.5
0	1	0	12	5	semilong	–
1	0	0	12	5	semilong	–
1	1	2	21	6	chiral	7
1	1	0	24	6.29	none	8.29
1/2	3/2	1	26.25	6.5	semilong	–
3/2	1/2	1	26.25	6.5	semilong	–

Table 4: *Lowest dimensional operators from vector multiplet IV.*

To summarize this appendix we include a table of the lowest dimension operators found above and their dimensions, as well as the form of the corresponding operators in the field theory when it is known. The first irrelevant operator is the top component of a long multiplet and contributes to the Kähler potential, but its exact form is not known.

Δ	j	l	$\|r\|$	Multiplet	Type	Operator
2.5	1/2	1/2	1	I	chiral	$S_1 = \int d^2\theta \ \mathrm{Tr}(AB)$
4	1	1	2	I	chiral	$S_2 = \int d^2\theta \ \mathrm{Tr}[(AB)^2]$
4	0	0	0	IV	chiral	$\Phi_0 = \int d^2\theta \ \mathrm{Tr}(W_1^2 + W_2^2)$
4	0	0	0	III	chiral	$\Psi_0 = \int d^2\theta \ \mathrm{Tr}(W_1^2 - W_2^2)$
5.29	1	1	0	I	long	$\mathcal{O}_1 = \int d^4\theta \ (?)$

Table 5: *Lowest dimensional operators.*

Appendix B. The moduli space of the deformed theory

In this appendix we consider deforming the superpotential of the Klebanov-Witten theory [10] by the relevant and marginal operators S_1 and S_2 given in (3.2), (3.3). The resulting moduli space is analyzed and shown to be of lower dimension than the original symmetric product of N copies of the conifold. In the dual gravity description this must be due to a deformation exerting a force on the D-branes. Such a deformation is forbidden by the equations of motion in our construction, so we conclude that these operators are not turned on. For the case of a Klebanov-Strassler background the conclusion should remain the same.

In the $SU(N) \times SU(N)$ theory with superpotential $W = h\epsilon_{ik}\epsilon_{jl}\mathrm{tr}(A^i B^j A^k B^l)$ the F-term equations require that the chiral fields $A_i, B_i \quad i = 1, 2$ will commute, so that they can be simultaneously diagonalized by gauge transformations. The D-term equations then lead to the general solution being N copies of the conifold. This branch describes D-branes moving separately on the 6 dimensional geometry. Such a branch must also exist for the deformed theory due to the no force condition on the D-branes, and we expect that the subspace of diagonal matrices that solve the F-term and D-term equations should give us the N'th symmetric product of the deformed 6 dimensional geometry.

For diagonal matrices the equations for the $N \times N$ matrices become decoupled so we can consider them as N identical equations for single fields. In this case the moduli space will be the solution to the F-term equations divided by the complexification of the $U(1)$ gauge group, and the original superpotential can be ignored. Since all fields are charged under the $U(1)$ we get

$$\mathrm{Dim(moduli\ space)} = \mathrm{Dim(F-term\ solutions)} - 2. \tag{B.1}$$

Since the dimension should be six, we get that the solutions to the F-term equations should form an 8 dimensional space.

We begin by considering the relevant chiral operator S_1. The general deformation of the superpotential is given by:

$$\Delta W = \lambda_{ij} \mathrm{Tr}(A_i B_j), \tag{B.2}$$

where λ_{ij} is constant matrix. The F-term equations for 1×1 scalars are:

$$\lambda \cdot B = 0, \qquad \lambda^t \cdot A = 0, \tag{B.3}$$

where we consider A, B as 2-vectors and λ as a 2×2 matrix. For $\det(\lambda) \neq 0$ there is no solution to the system of equations. For $\det(\lambda) = 0$, $\lambda \neq 0$ (so $\mathrm{rank}(\lambda) = 1$), there is a 2 dimensional space of complex solutions so the moduli space is 2 dimensional. Only for $\lambda = 0$ we get a moduli space large enough for describing free D-branes.

We now add also the marginal operator S_2. The deformed superpotential is

$$\Delta W = \lambda_{ij} \mathrm{Tr}(A_i B_j) + \frac{1}{2} \sigma_{ijkl} \mathrm{Tr}(A_i B_j A_k B_l), \tag{B.4}$$

with $\sigma_{ijkl} = \sigma_{kjil} = \sigma_{ilkj}$. The symmetry condition for the indices comes from the fact that the chiral marginal operator is the $j = l = 1$ combination of the four fields. From the cyclicity of the trace we also get $\sigma_{ijkl} = \sigma_{klij}$.

The F-term equations for 1×1 scalars are now

$$\begin{gathered} \lambda_{ij} B_j + \frac{1}{2} \sigma_{ijkl} B_j A_k B_l + \frac{1}{2} \sigma_{kjil} B_j A_k B_l = \lambda_{ij} B_j + \sigma_{ijkl} B_j A_k B_l = 0, \\ \lambda_{ji} A_j + \sigma_{jikl} A_k B_l A_j = 0, \end{gathered} \tag{B.5}$$

where we used the symmetries of σ. There are 4 complex fields in the equations so the maximal dimension for the space of solutions is 8. If the solutions indeed form an 8

dimensional space then for a generic solution any change $A_i \to A_i+\delta A_i$ and $B_i \to B_i+\delta B_i$ will result in a new solution. Taking the first equation and shifting only the A_i fields we find for any δA_i

$$\begin{aligned}\lambda_{ij}B_j+&\sigma_{ijkl}B_jA_kB_l+\sigma_{ijkl}B_j\delta A_kB_l=0\\ &\Rightarrow \sigma_{ijkl}B_j\delta A_kB_l=0\\ &\Rightarrow \sigma_{ijkl}B_jB_l=0.\end{aligned} \tag{B.6}$$

Again this holds for all solutions so we can now shift B_i to find

$$\begin{aligned}\sigma_{ijkl}B_jB_l+&\sigma_{ijkl}\delta B_jB_l+\sigma_{ijkl}B_j\delta B_l+\sigma_{ijkl}\delta B_j\delta B_l=0\\ &\Rightarrow (\sigma_{ijkl}+\sigma_{ilkj})B_j\delta B_l=0\\ &\Rightarrow \sigma_{ijkl}+\sigma_{ilkj}=0\\ &\Rightarrow \sigma_{ijkl}=0.\end{aligned} \tag{B.7}$$

Where in the last step we used the symmetry properties of σ_{ijkl}. Hence we are left with only the relevant deformation which also vanishes by the previous argument.

This argument can be generalized to any higher deformation of this form. Consider the deformation $\mathrm{Tr}(A_{i_1}B_{j_1}\cdots A_{i_n}B_{j_n})$. Since the $SU(2)\times SU(2)$ representation is $j=l=\frac{n}{2}$ the coefficient of this term is symmetric under exchange of the j indices and under exchange of the i indices. We can then carry out a similar argument, where we take at each step another derivative with respect to A or B. Due to the symmetry property, each time we will get the same coefficient with one lower power of the fields. After $2n$ steps we will be left with only this term and no lower terms, and will arrive to the conclusion that the coefficient must vanish.

We conclude that turning on these types of deformations the diagonal branch of the moduli space cannot have 6 dimensions. Since the geometry discussed in section 2 accommodates D-branes on a 6 dimensional space, these operators are not turned on by deforming the Klebanov-Strassler solution to a compact Calabi-Yau. In particular the relevant and marginal deformations ($n=1$ and $n=2$) are not turned on.

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